# VECTOR THEORY OF SELF-FOCUSING

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The propagation of light beams in nonlinear media is usually described in a scalar parabolic approximation. For some models of a nonlinear medium, the beam width can become as small as desired during its development. In that case, it is necessary to give up the scalar theory and to solve the complete system of Maxwell equations. Such a formulation of the problem is very complex and therefore extremely few papers [1-4] have been devoted to the vector theory of self-focusing. The present paper presents the results of a study of vector self-focusing for the simplest case of the lowest axially symmetric TM mode.

The system of stationary nonlinear Maxwell equations

$$\mathrm{rot}\,\mathbf{H} = -ik_0\varepsilon(|\mathbf{E}|^2)\mathbf{E}, \tag{1}$$

$$\mathrm{rot}\,\mathbf{E} = ik_0\mathbf{H},\ k_0 = \omega/c \tag{2}$$

for a monochromataic beam propagating along the z axis and of bounded cross section has two integrals of motion independent of z:

$$I_1 = \int\int dxdy\,\{[\mathbf{E}, \mathbf{H}^*]_z + \text{c.c.}\},$$

$$I_2 = \int\int dxdy\left\{\frac{1}{ik_0}\left(\left[\mathbf{E}, \frac{\partial \mathbf{H}^*}{\partial z}\right]_z + \left[\frac{\partial \mathbf{E}^*}{\partial z}, \mathbf{H}\right]_z\right) + \Phi\right\} =$$

$$= \int\int dxdy\,\{\varepsilon(|E_z|^2 - |E_x|^2 - |E_y|^2) + |H_z|^2 - |H_x|^2 - |H_y|^2 + \Phi\},$$

where $\Phi(|\mathbf{E}|^2) = \int\limits_0^{|\mathbf{E}|^2} \frac{d\varepsilon}{d\eta}\eta d\eta$. Consequently the integral over the entire cross section of the beam of the z component of the Poynting vector is expressed through $I_1$ in the form $I_Z = (c/16\pi)I_1$ and replaces the well-known integral $I_0 = \int\int dxdy|\mathbf{E}|^2$ for the scalar parabolic equation [5, 6].

As is well known, the stability of stationary solutions of the scalar wave equation is determined by the sign of the derivative of $I_0$ with respect to the propagation constant [7]. One can expect that the stability of stationary solutions of the complete system of Maxwell equations (1) and (2) will be determined correspondingly by the derivative of the integral $I_Z$ with respect to the propagation constant.

To check this hypothesis, the lowest axially symmetric TM mode was studied for the case of a cubic medium with a dielectric constant $\varepsilon = \varepsilon_0\{1 + \varepsilon_2|\mathbf{E}|^2\}$. Note that the analysis of vector self-focusing in [1], which was based on the dependence of the quantity $I = \int\int dxdy|\mathbf{E}|^2$ on beam width, is incorrect since I is not an integral of motion for the equation system (1), (2).

Writing the intensity of the electric field in the form $\mathbf{E} = (\gamma/\sqrt{\varepsilon_2})\mathbf{A}\exp[i(kz - \omega t)]$ and eliminating the magnetic field from the original equations, we obtain the following system of equations for the amplitude **A** of the fundamental TM mode:

$$\frac{\partial^2 A_r}{\partial\tau^2} - 2i\frac{\partial A_r}{\partial\tau} + \gamma\frac{\partial^2 A_z}{\partial\rho\partial\tau} + i\gamma\frac{\partial A_z}{\partial\rho} = \gamma(|A_r|^2 + |A_z|^2)A_r; \tag{3}$$

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A_r}{\partial\rho}\right) + \frac{i}{\rho}\frac{\partial}{\partial\rho}(\rho A_r) - \frac{\gamma}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial A_z}{\partial\rho}\right) = \gamma\left(\frac{1}{\gamma^2} + |A_r|^2 + |A_z|^2\right)A_z, \tag{4}$$

where $\tau = kz$, $\rho = \gamma kr$ are dimensionless variables; $k = k_0\sqrt{\varepsilon_0}$; $\gamma$ is a free parameter which is the propagation constant and which determines the effective width of the field distribution.

Stationary distributions of the field satisfy the system of equations which is obtained by substitution of

$$A_j(\rho, \tau) = \widetilde{A}_j(\rho)\exp[i(\delta\tau + \delta_j)],\ j = r, z,$$

in Eqs. (3), (4), where $\delta = \sqrt{1+\gamma^2} - 1$ and $\delta_r - \delta_z = \pi/2$.

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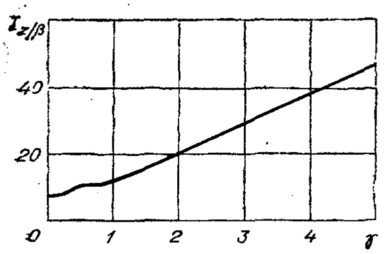

Fig. 1

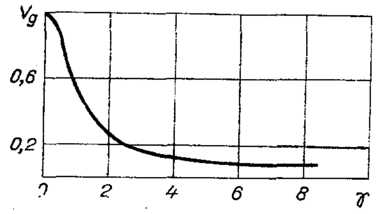

Fig. 2

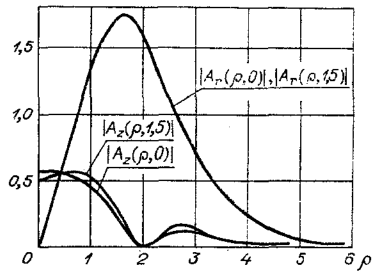

Fig. 3

The amplitude $\tilde{A}_j(\rho)$ satisfies the boundary conditions

$$\tilde{A}_r(0) = \tilde{A}_r(\infty) = 0, \ (d\tilde{A}_z/d\rho)(0) = \tilde{A}_z(\infty) = 0. \tag{5}$$

The dependence of the integral $I_z$ and of the energy transport velocity $V = I_z/W$ on the parameter $\gamma$, where $W = (1/8\pi)\iint dxdy\{|\mathbf{E}|^2 + |\mathbf{H}|^2\}$, was calculated for a stationary distribution of the fundamental TM mode. As is clear from the dependence of the quantity $I_z/\beta$ on $\gamma$, where $\beta = c^3/(4\omega^2\varepsilon_2\sqrt{\varepsilon_0})$, shown in Fig. 1, the energy flux transported by the mode in the z direction increases as $\gamma$ increases (i.e., with reduction in width of the mode). Consequently, one can expect that the corresponding mode in a cubic medium would be stable. The dependence of the quantity $V_g = V/(c/\sqrt{\varepsilon_0})$ on $\gamma$ is shown in Fig. 2. The velocity of energy transport decreases as the width of the mode decreases, which is in agreement with results known from waveguide theory [8].

The stability of the fundamental TM mode with respect to small perturbations was also studied by numerical solution of the equation system (3), (4) in which the quasioptical approximation $|\partial^2 A_r/\partial\tau^2| \ll |\partial A_r/\partial\tau|$ was used for sufficiently small values of $\gamma$. The resultant equations were approximated by an implicit three-level symmetric regular finite-difference scheme of second order over both variables. After proper transformation of the boundary conditions at infinity [9], the resultant system of algebraic equations was solved by the stepping method. Monitoring of the accuracy of the calculations was accomplished through conservation of the integral $I_z$. The relative accuracy of the conservation was 3%. As an initial distribution in this quasioptical approximation, a perturbed field distribution for the fundamental TM mode was selected:

$$E_r(\rho, 0) = \tilde{A}_r(\rho)\exp(i\delta_r) + ia\rho\exp(-a_1\rho^2),$$
$$E_z(\rho, 0) = \tilde{A}_z(\rho)\exp(i\delta_z) + b\exp(-b_1\rho^2).$$

The quantities $E_r$ and $E_z$ satisfied the boundary conditions (5). The values of the parameters $a$, $a_1$, b, and $b_1$ were chosen so that the relative variation of the integral $I_z$ was 1-20%.

Numerical calculations on a BÉSM-4 computer showed that the transverse distribution during development remained close to stationary while at the same distances and for the same relative perturbations of the amplitudes, the fundamental mode in the scalar quasioptical approximation either collapsed or became diffuse [6].

Figure 3 shows the radial distributions $|A_r(\rho, \tau)|$ and $|A_z(\rho, \tau)|$ for $\gamma = 0.2$, $a = -0.05$, $a_1 = 0.2$, $b = 0.01$, $b_1 = 0.1$, and various values of $\tau$. The relative contribution of perturbations to the value of $I_z$ was 5%. In practice, the specified parameters correspond, for example, to a beam from a ruby laser having a diameter $\approx 10\lambda$, a power of 40 kW, and propagating in a medium with $\varepsilon_2 = 1.8 \cdot 10^{-11}$ absolute units.

In the general case of vector self-channeling cylindrical waveguides [2],

$$I_z/\beta = \int_0^\infty \left\{\sqrt{1+\gamma^2}\,(\tilde{A}_r^2 + \tilde{A}_\varphi^2) - \gamma\left(\tilde{A}_r\frac{d\tilde{A}_z}{d\rho} + \frac{m}{\rho}\tilde{A}_\varphi\tilde{A}_z\right)\right\}\rho d\rho > 0,$$

where m is the azimuthal index. Based on the continuity of stationary solutions in terms of $\gamma$, one can show that $I_z \sim \gamma^2$ when $\gamma \to 0$ and $I_z \sim \gamma$ when $\gamma \to \infty$. Therefore, $dI_z/d\gamma > 0$ in these cases of greatest practical interest and the formation of singularities evidently does not occur.

Thus, the study pointed to the possibility of experimental observation of stationary vector waveguide solutions in cubic media.

## LITERATURE CITED


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# PLASMA HEATING AT CONSTANT IMPEDANCE

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It is well known that the plasma conductivity $\sigma$ depends strongly on the temperature T [1], $\sigma \sim T^{3/2}$, which leads to breakdown in plasma matching during heating with an energy source and to a drop in heating efficiency. Constancy of impedance facilitates broadband matching of an energy source with a target [2, 3]. This paper demonstrates that the impedance changes little during pulsed heating of a solid plasma through propagation of an ionization wave [4].

We consider a solid dielectric between the two conductors $S_1$ and $S_2$ of a transmission line (Fig. 1). A thin wire or film AB is within the dielectric. We limit ourselves to the simplest case where the conductors $S_1$ and $S_2$ are plane-parallel plates. A powerful radio or video pulse is fed into the line [5, 6], the film explodes [7, 8], and an ionization wave is propagated from the film with the field and current pattern shown in Fig. 2. The ionization front is propagated to the left, $E_1 \neq 0$ on the left ahead of the front, $\sigma_1 = 0$ in the dielectric, and $\sigma = \sigma_2$ on the right behind the front. The uhf field or short pulse does not penetrate within the conducting plasma behind the ionization front ($E_2 = 0$) so that the pulsed current j is zero everywhere except for a thin skin layer in which energy is deposited, and the propagation of the discharge, as noted in [4], is completely analogous to the detonation process [4, 9]. In the system shown in Fig. 1, propagation of both a breakdown wave and an ionization wave is possible with the wave having the greater velocity being the one propagated [4].

The propagation of ionization waves in gases was discussed in detail in [4] and the propagation of ionization waves was first discussed in [10, 11]. The present paper studies the features of ionization-wave propagation at condensed-state densities.

For the velocity D of a plane detonation wave and the specific internal energy $\varepsilon$ of the material behind the front, the relations [4, 9]

$$D = [2(\gamma^2 - 1)(S/\rho)]^{1/3}, \qquad (1)$$

$$\varepsilon = \frac{2^{2/3}}{(\gamma^2 - 1)^{1/3}(\gamma + 1)}(S/\rho)^{2/3} = \frac{\gamma}{(\gamma^2 - 1)(\gamma + 1)} D^2 \qquad (2)$$

are valid, where S is the flux of absorbed energy, erg/(sec · cm$^2$); $\rho$ is the density of the material; $\gamma$ is the effective adiabatic index [9].

For example [7], let the pulse energy be 10 kJ = $10^{11}$ ergs, the duration $\tau = 10^{-9}$ sec, which corresponds to a power of $\sim 10^{13}$ W = $10^{20}$ erg/sec, let the heated sample be a cylinder of radius $r_0 = 1$ mm and length 2 mm with the lateral surface of the cylinder $\sim$10 mm$^2$ or $\sim$0.1 cm$^2$, and $S \simeq 10^{14}$ W/cm$^2$ = $10^{21}$ erg/(sec · cm$^2$). One can set $\rho \sim 1$ g/cm$^3$ for a solid dielectric.

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